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Subject: **Physics**

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Paper No. : Nuclear and Particle Physics

Module : Nuclear Models -2

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Description of Module	
Subject Name	Physics
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1. Applications of Liquid Drop Model

To go further, we are interested in the deviations between the experimental measured values and the values calculated from liquid drop model for the binding energies, and in understanding these deviations in terms of microscopic models. The deviation between the measured values and the Bethe-Weizsacker values for binding energy per nucleon is given in fig. 1. While the fit is generally good, periodic strong deviations are observed at certain proton or neutron numbers. These deviations indicate additional stability and provide evidence for shell closures at nucleon numbers 2, 8, 20, 28, 50, 82 and 126 (neutrons).

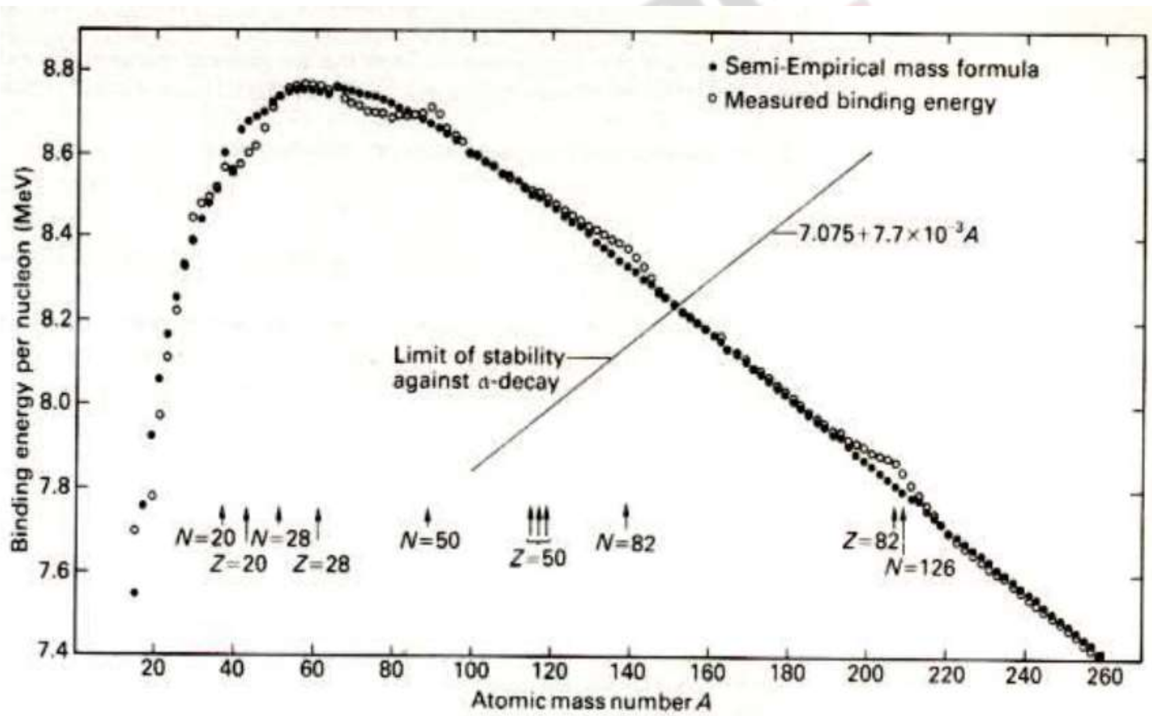

Fig. 1: Difference between measured masses values and those predicated by the Bethe-Weizsacker formula for Binding energy per nucleon (B.E./A

1.1 Interpretation of Liquid Drop Mass Formula

The liquid drop mass equation leads to the following general conclusions

- All nuclei except three to four exceptions are most stable in nature up to lead. Nuclei near ^{56}Fe are the most stable in Nature, which can be proved by differentiating semi-empirical mass equation with respect to Z and A and setting the result equal to zero to obtain the minimum. This is the point at which the competition between surface energy losses and Coulomb energy losses is balanced, corresponding to the peak in the average binding energy curve. Therefore, all nuclei are thermodynamically driven to ^{56}Fe.
- Nuclei with low A lose binding energy primarily due to the surface energy term; i.e. as nuclei become smaller, a larger fraction of the nucleons are on the surface. By adding two light nuclei together in a nuclear fusion reaction, a more stable system is formed and energy is released. This is the principle upon which stars generate energy and is the basis for the nuclear fusion reactor.
- Nuclei with high A lose binding due to their large Coulomb energy. In heavier nuclei due to the presence of large number of protons large coulomb repulsion exist between them and coulomb energy dominates over surface energy and splitting take place by releasing excess energy. Thus, splitting a heavy nucleus in a nuclear fission reaction leads to more stable products and energy is released**.**
- The symmetry term favors N/Z = 1. So all the stable nuclei lie on or near a line called stability line in *Z*, *N* plane. For light nuclei this is observed up to Ca^{40}. However, for heavier nuclei neutron and proton ratio is found to be greater than unity i.e. N/Z > 1. This trend is observed due to additional number of neutrons to balance with the Coulomb energy; i.e. larger A for a given Z lowers the Coulomb energy loss.
- Except few exceptions, all stable nuclei have $N = Z$, $Z+1$, or $Z+2$ upto mass $A = 35$ & thereafter increases faster than Z until in lead $N \approx 1.5\ Z$.
- Nuclei with even numbers of neutrons and protons have higher binding energy relative to those with an odd nucleon.

2. Stability of Nucleus

In the plot of N versus Z graph for the most stable nuclei shows the divergence from N = Z, as preferred by the symmetry term in the liquid drop (LD) mass equation. To trace the evolution of this plot, it is useful to examine the prediction of the mass formula for individual isobars A, which are connected via beta decay. In order to find out the stability of the nucleus, we can take the first derivative with respect to Z to calculate the optimal Z such that mass is minimum.

From the Bethe-Weizsacker Mass formula

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} \pm \delta + \eta$$

On rearranging the above the equation

$$M(A,Z)\,c^2 = \alpha A + \beta Z + \gamma Z^2 \mp \delta - \eta$$

This is the equation of parabola in Z for given A, and the minimum of the parabola corresponds to the most stable nucleus for a given isobar Z, which can be obtained by differentiating above and setting the result equal to zero. These minima define the 'valley of beta stability'. Where

$$\alpha = M_n c^2 - a_V + a_A + \frac{a_S}{A^{1/3}}$$

$$\beta = -4a_A - (M_n - M_p)c^2 \approx -4a_A \quad \text{and} \quad \gamma = \frac{4a_A}{A} + \frac{a_C}{A^{1/3}}$$

A nucleus of mass *A* is most stable with Z_0 protons, when mass is minimum, so differentiating above equation wrt to Z and putting it to equal to zero-

$$\frac{\partial}{\partial Z}(M\,c^2) = 0 \qquad \Rightarrow \quad \beta + 2\gamma Z_0 = 0 \quad \text{or} \qquad Z_0 = -\frac{\beta}{2\gamma}$$

i.e.

$$Z_0 = -\frac{\beta}{2\gamma} \approx \frac{A/2}{1 + \frac{1}{4}(a_C / a_A) A^{2/3}}$$

2.1 Implications

From the above equation we can conclude that the most stable nuclei of various mass numbers *A* is determined by the value of Z_0. The deviation of stability line from the condition N=Z or Z= A/2 is due to competition between *the Coulomb energy*, which favors $Z_0 < A/2$, & *the Asymmetry energy* which favors $Z_0 = A/2$, and is represented in fig. 2.

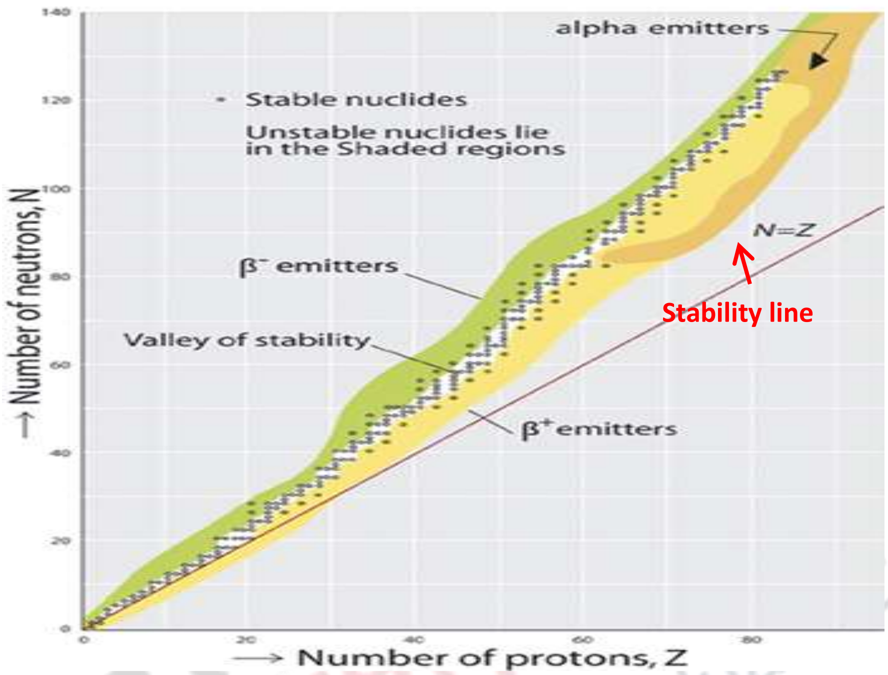

Fig. 2: Deviation of Stability line with increasing number of protons

Because of the pairing energy term δ in the mass equation, the solution fall into two categories according to whether even-A or odd-A nuclei is taken into consideration. For odd-A nuclei the paring term (δ) is zero, leading to a single parabola for both e-o and o-e nuclei shown in fig. 3(a). The important result for this case is that **there is only one beta-stable isotope for odd-A nuclei**.

For even-A nuclei, the pairing term can be either +1 for e-e nuclei or -1 for o-o nuclei. This yields two parabolas for the same mass number shown in fig. 3 (b), one for even Z (e-e) and the other for odd Z (o-o). Since the beta decay of an e-e nucleus produces an o-o nucleus, the decay chain alternates between the two parabolas. For even isobars, since the values of δ can be positive or -negative, so the mass equation gives two parabolas, differing in mass by 2δ. The important result for this case is that **even-A nucleus can have up to three beta-stable isobars, depending on the relative positions of the two parabola**.

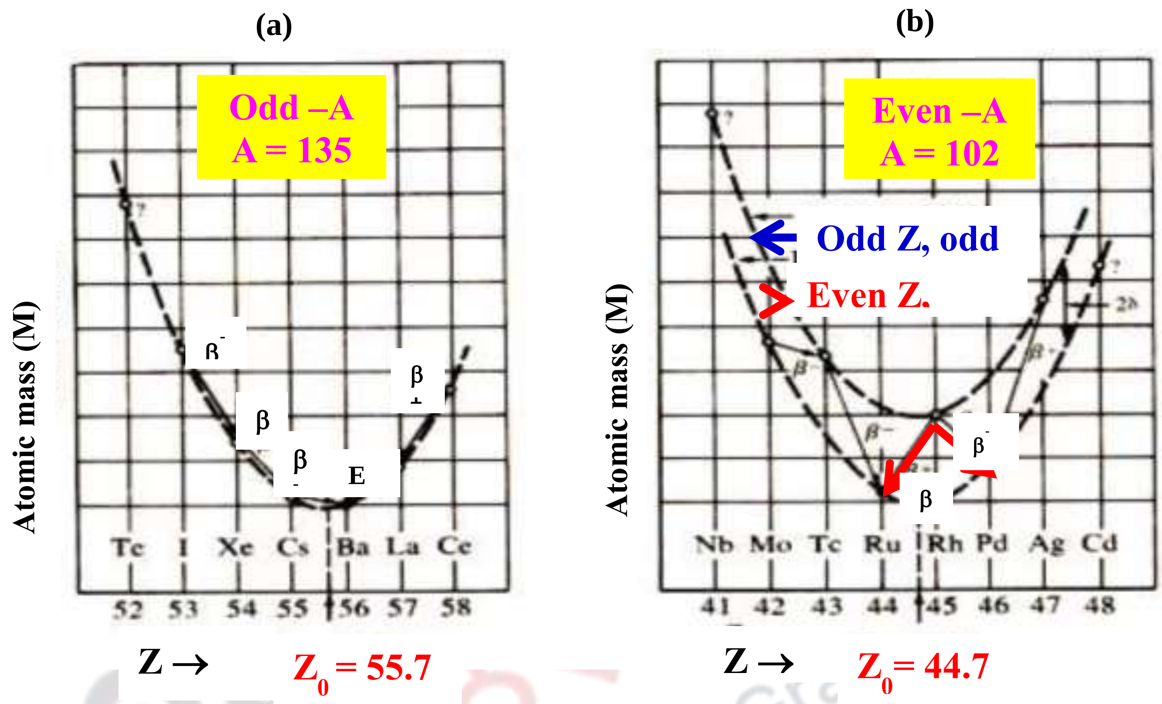

Fig. 3: Mass parabolas for Even and Odd A. Z_0 is showing position of minima, corresponding to most stable isobar.

In the above fig. 3 (a) is shown the plot of M (A, Z) against Z for odd A isobars with A= 135. This is a parabola for which the lowest point is at Z_0= 55.7. The stable isobar that is actually observed at this mass number is Ba^{135} for which Z= 56. The nuclides falling on either side of the stable isobar are all unstable. Those on the lower Z side (Z <56) are β^- active while those on the higher side (Z > 56) are β^+ active or electron capturing. Each of these nuclei undergoes β- transformation into the product nucleus with Z one unit higher or lower respectively as shown in the figure. Such transformation goes on step by step till the stable end product is reached.

Fig. 3 (b) shows the two mass parabolas for the even A isobars with A = 102. The upper one is for odd Z, odd N isobars, while the lower one is for the even Z, even N isobars. The most stable isobar in this case falls on the lower parabola. On using the equation for Z_0 to calculate the minimum value of Z_0 protons for stable isobar, we obtained Z_0 = 44.7. Actually a stable nuclide ^{102}Ru at Z = 44 is observed at this mass number. In addition to this, another stable e-e nuclide ^{102}Pd (Z =46) also exist at this mass number A. The two stable isobars differ in Z by two units. The o-o isobar ^{102}Rh with Z = 45 between these two falls on the upper parabola and has an atomic mass greater than those of either of the above two. Hence ^{102}Rh is not stable. It shows both β^+ and β^- activities. β^+ emission transforms it to ^{102}Ru while by β^- emission it transforms to ^{102}Pd. Fig. 3(b) also show two other o-o isobars ^{102}Tc with Z= 43 and ^{102}Ag with Z = 47 on the upper parabola. Their atomic masses are higher than those of the neighboring e-e isobars ^{102}Ru and ^{102}Pd respectively. Hence none of those latter two isobars can be β^- active. β-transformation changes Z by one unit.

Depending on the curvature of the parabolas & separation 2δ, there can be several stable even-even isobars for which Z differs by two units. There exist a possibility that for certain odd-odd nuclei both conditions are fulfilled so both β^- & β^+ *decay* from the identical nuclide are possible and do indeed occur as shown in fig. 4.

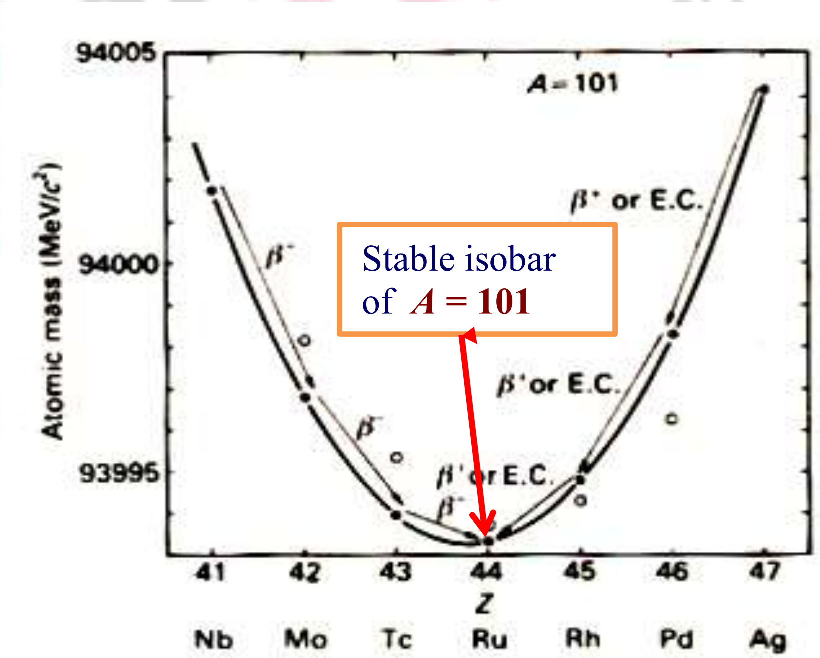

Fig. 4: Figure indicating stable isobar of odd-A nuclei

2.2 Rules of decay for even- A nuclei

There are certain rules by which an even-A nuclei can decay

Even-*A* & **Even-*Z*** nuclei have a high B.E. due to pairing term, whereas **odd-*Z*** nuclei have a lower B.E. due to opposite contribution from this term. Thus there are two curves of isobar mass against ***Z*** & alternate ***Z*** lie on different curves. The odd nuclei with $Z = 43$ can decay by Electron capture (EC) to $Z = 42$ or by negative beta β^- decay to $Z = 44$. The important conclusion of this results is that there are two stable isobars for $A = 100$, namely $Z = 42$ & 44, which is true as shown in fig. 5.

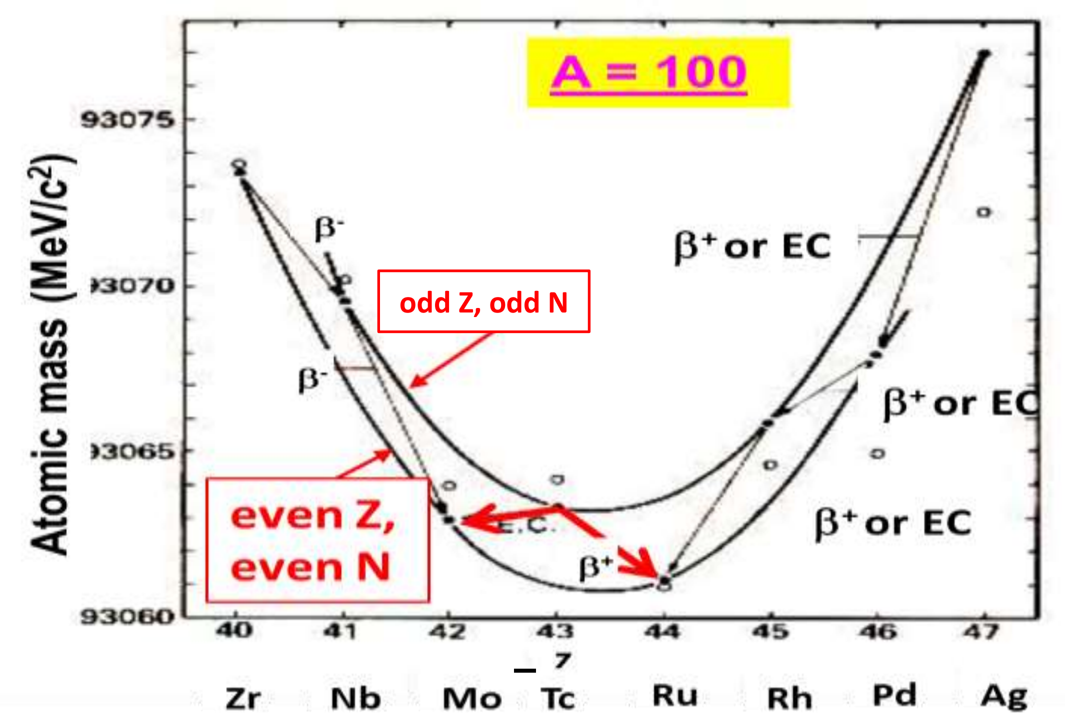

Figure 5: Figure showing two stable isobar corresponding to even-A nuclei

So we conclude from the above discussion that there can be two even-even stable isobars (A even) for which Z differs by two units.

Sometimes there is a possibility that even-A nucleus leaves an odd-odd isobar in an excited state, so there a probability that it energetically be able to decay by all β^- and β^+ (or E.C.) modes. ($^{40}K_{19}$) as shown in fig. 6.

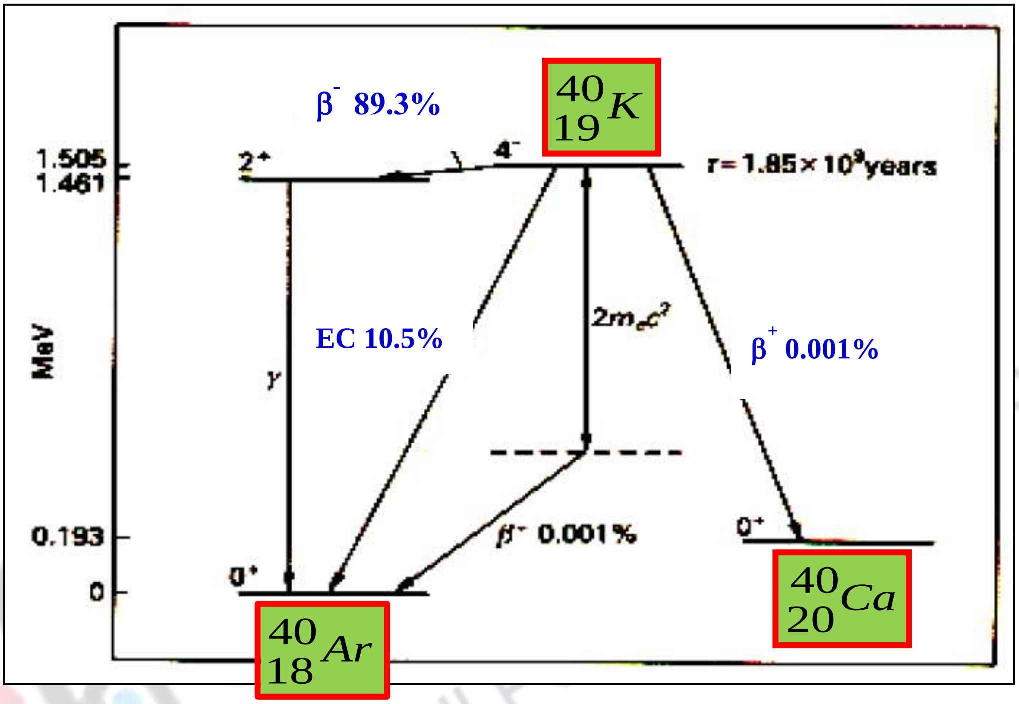

Fig. 6: Decay probability of even-A nucleus $^{40}K_{19}$. It shows that $^{40}K_{19}$ can decay by all three modes

Summary

The Bethe–Weizsacker mass formula is a phenomenological understanding of nuclear binding energies as function of A, Z and N. and explains the experimental B.E./A quite well. It explains the valley of stability; and explains the energetics of radioactive decays, fission and fusion. The Bethe–Weizsacker mass formula also define the limit of stability against alpha-decay and spontaneous fission. The Coulomb term and the Asymmetry energy term play an important role in deciding the stability of nuclei. The odd-A nuclei can have only one mass parabola due to paring term is zero, whereas even-A nuclei have two mass parabolas as pairing energy tern can take two values either $-\delta$ or $+\delta$.